

## DESIGN OF A PROPORTIONAL-INTEGRAL POWER SYSTEM STABILIZER

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## ABSTRACT

Stabilization of power systems is investigated using a proportional-integral (PI) power system stabilizer. Digital (sampled-data) PI stabilizers as well as analog (continuous-time) PI stabilizers are examined in this paper. Two approaches, viz., the root-locus method and the suboptimal regulator method, are presented for determining the optimal stabilizer gains of the proposed PI stabilizer.

The dynamic responses following a step disturbance by digital simulation are obtained by means of three types of stabilizers: the conventional power system stabilizer, the optimal stabilizer and the PI stabilizer. Simulation results show that the proposed PI stabilizer yields better system dynamic performance than the others in the sense of having greater damping in response to a step disturbance. A characteristic feature of the proposed PI stabilizer is that it is very simple for practical implementation, especially in the digital case, as commercial PI controllers have been widely employed by the industry for years.

## 1. INTRODUCTION

In the literature considerable efforts have been placed on the application of power system stabilizers (PSS) to improve the dynamic stability (small perturbation stability) of power system [1-10]. The power system stabilizer usually employed by the utility industry is a lead-lag network using the speed as input. The fundamental concept for the design of such a power system stabilizer is to compensate the phase lag resulting from the voltage regulator, exciter and generator so that a supplementary damping torque component which is in phase with rotor speed is generated [1]. This supplementary damping torque can be employed to enhance the dynamic stability of power system.

The purpose of this paper is to present a new approach for the design of a power system stabilizer. The proposed technique makes use of the theory of proportional-plus-integral (PI) controllers which are the most commonly used control algorithms in the process industry. The theory of PI controllers is simple, and there are good PI control algorithms available in the literature [11, 12]. Analysis of the phase angle of a PI controller reveals that the phase compensation required for a power system stabilizer can be achieved by a PI stabilizer. However, applications of PI controllers to the design of PSS have not yet been exploited. Malik, Hope and Badr [13] are, probably, the first to investigate a PID voltage regulator for a synchronous machine.

With the advancement in microprocessor technology a digital (sampled-data) power system stabilizer using analog-digital converters and microprocessors to implement the lead-lag type analog (continuous time) stabilizer was proposed [14]. An advantage of the sampled-data stabilizer is that in addition to providing damping effect to the system it also has the capability of incorporating self-checking and diagnostic operation in case of malfunction. Thus higher reliability can be achieved by using a digital stabilizer.

In this paper, both the analog and digital power system stabilizers will be examined. Two approaches, viz., the root-locus method and the suboptimal regulator method, are developed for the design of PI power system stabilizers. Examples for a single machine infinite bus system and a multimachine system are given to illustrate the effectiveness of the proposed PI stabilizer and simulation results are presented and compared with those obtained by using the conventional and the optimal stabilizers. The main features of the proposed PI stabilizer are as follows:

- (i) The proposed PI stabilizer, especially in the digital case, is relatively simpler for practical implementation than the conventional stabilizer and optimal stabilizer.
- (ii) The PI stabilizer yields better system dynamic performance than that obtained by using the conventional lead-lag stabilizer in that the step responses have a shorter settling time and greater damping effect. Dynamic stability of the power system can, therefore, be greatly improved by the PI stabilizer.
- (iii) Two systematic procedures for the design of PI stabilizer are proposed in the paper.

## 2. FUNDAMENTAL CONCEPTS OF PI POWER SYSTEM STABILIZER

As first pointed out by deMello and Concordia [1] and later explained in details by Larsen and Swann [15], the fundamental concepts for the design of a conventional lead-lag power system stabilizer is to compensate the phase-lag resulting from the voltage regulator, the exciter and the synchronous generator. In other words, for a power system stabilizer employing the speed as input signal, the following condition must be met:

$$\angle G(s) + \angle GEP(s) = 0 \quad (1)$$

where  $GEP(s)$  is transfer function of the generator, exciter and power system and  $G(s)$  is transfer function of the power system stabilizer. The phase angle of  $GEP(s)$  is a function of the operating condition of the power system as well as the parameters of the excitation systems. Therefore, the phase angle of  $G(s)$  should be adjustable in the range of  $0^\circ$  to  $360^\circ$  in order that eqn. (1) could be satisfied all the time.

Next, let's consider the proposed PI power system stabilizer. The transfer function of this stabilizer is given by

$$G(s) = \frac{u(s)}{\Delta\omega(s)} = \left( K_p + \frac{K_I}{s} \right) \quad (2)$$

where  $K_p$  is the gain of the proportional stabilizer and  $K_I$  is the gain of the integral stabilizer. Fig. 1 gives the phase angle of  $G(s)$  for different values of  $K_p$ 's and  $K_I$ 's. It is clear that a phase angle from  $0^\circ$  to  $360^\circ$  in the  $s$  plane could be obtained by employing a PI stabilizer of which the transfer function is given in eqn. (2). Therefore, the PI power system stabilizer is justified from the frequency-domain consideration.

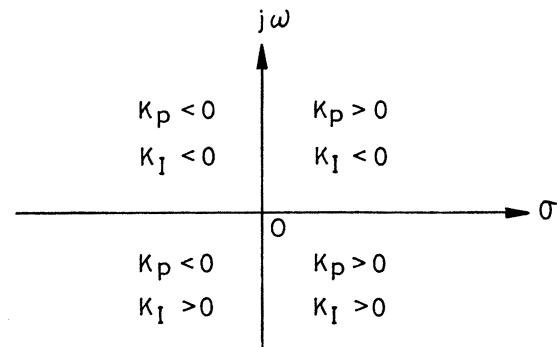
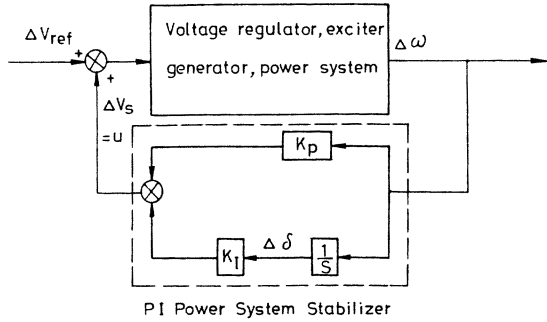


Fig. 1 Phase angle of  $G(s) = K_p + \frac{K_I}{s}$

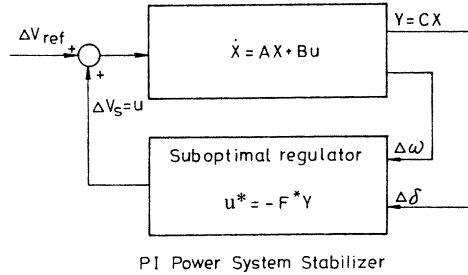
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## 3. PI POWER SYSTEM STABILIZER DESIGN USING THE SUBOPTIMAL REGULATOR METHOD

In this section, the theory of suboptimal regulator will be employed to determine the optimal stabilizer gains  $K_p$  and  $K_I$  of the proposed PI power system stabilizer. Both the analog and digital PI stabilizers will be examined.



(a) Block diagram of the PI Power System Stabilizer



(b) suboptimal regulator

Fig. 2. PI stabilizer design formulated as a suboptimal regulator problem

#### A. The Analog PI Power System Stabilizer

Fig. 2 illustrates how the problem of the determination of a set of stabilizer gains  $K_p$  and  $K_I$  of the PI stabilizer can be formulated as a suboptimal regulator problem. The state equation of the system is first written as

$$\dot{X}(t) = A X(t) + B U(t) \quad (3)$$

$$Y(t) = C X(t) \quad (4)$$

where  $X$  is the state vector,  $U$  is the control vector,  $Y$  is the output vector and  $A$ ,  $B$  and  $C$  are constant matrices. Since the input signal of the PI stabilizer is the speed ( $\Delta\omega$ ) of which the integral is the torque angle ( $\Delta\delta$ ), we need two types of state variables ( $\Delta\omega$  and  $\Delta\delta$ ) as the input signals of the suboptimal regulator.

The performance index to be minimized is given by

$$J = \frac{1}{2} \int_0^{\infty} [X^T(t) Q X(t) + U^T(t) R U(t)] dt \quad (5)$$

where  $Q$  is a positive semi-definite matrix and  $R$  is a positive definite matrix. Then, the optimal stabilizing signal  $U^*$  is given by [16–18]:

$$U^*(t) = -F^* Y(t) \quad (6)$$

where

$$F^* = R^{-1} B^T K^* L^* C^T [CL^* C^T]^{-1} \quad (7)$$

$$O = K^* A^* + A^{*T} K^* + Q + C^T F^{*T} R F^* C \quad (8)$$

$$O = L^* A^{*T} + A^* L^* + I \quad (9)$$

$$A^* = A - B F^* C \quad (10)$$

#### B. The Digital PI Power System Stabilizer

In the design of a digital PI power system stabilizer, the state equations (3) and (4) are first converted to a set of difference equations:

$$X(n+1) = \phi X(n) + \theta U(n) \quad (11)$$

$$Y(n) = C X(n) \quad (12)$$

where

$$\phi = e^{A T_s} \quad (13)$$

$$\theta = \int_0^{T_s} \phi(T-\tau) B d\tau \quad (14)$$

and  $T_s$  is the sampling period.

The performance index to be minimized is

$$J = \frac{1}{2} \sum_{n=0}^{\infty} [X^T Q X + U^T R U] \quad (15)$$

Then, the optimal stabilizing signal is found to be [19]:

$$U^*(t) = U^*(n) \quad n T_s \leq t < (n+1) T_s \quad (16)$$

and

$$U^*(n) = -F^* Y(n) \quad (17)$$

where

$$F^* = R^{-1} B^T K^* \phi_c^* L^* C^T [CL^* C^T]^{-1} \quad (18)$$

$$K^* = \phi^{*T} K^* \phi^* + Q + C^T F^{*T} R F^* C \quad (19)$$

$$L^* = \phi^* L^* \phi^{*T} + I \quad (20)$$

$$\phi_c^* = \phi^* + B F^* C \quad (21)$$

#### 4. EXAMPLE 1: SINGLE MACHINE INFINITE BUS SYSTEM

##### A. The Analog Power System Stabilizer

Consider a single machine infinite bus system as shown in Fig. 3. The linearized incremental model of this system with the voltage regulator and the exciter included is shown in Fig. 4 [1, 2]. The system parameters are given as follows [2, 20]:

$$\begin{aligned} K_1 &= 1.4479 & K_2 &= 1.3174 \\ K_3 &= 0.3072 & K_4 &= 1.8050 \\ K_5 &= 0.0294 & K_6 &= 0.5257 \\ K_A &= 400 & T_F &= 1.0 \\ T_A &= 0.05 & D &= 0 \\ T_{d_0} &= 5.9 & K_E &= -0.17 \\ M = 2H &= 4.74 & T_E &= 0.95 \\ K_F &= 0.025 \end{aligned}$$

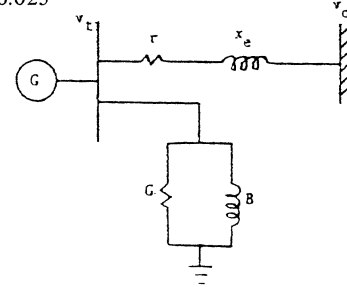


Fig. 3. System configuration for a single machine connected to a large power system through an external impedance

The state vector and output vector are chosen to be

$$X = [\Delta e'_q, \Delta e_{FD}, \Delta V_A, \Delta V_F, \Delta\delta, \Delta\omega]^T$$

$$Y = [\Delta\delta, \Delta\omega]^T$$

The system matrices are

$$A = \begin{bmatrix} -0.5517 & 0.1695 & 0 & 0 & -0.3060 & 0 \\ 0 & 0.1789 & 1.0526 & 0 & 0 & 0 \\ -4205.6 & 0 & -20 & -8000 & -235.2 & 0 \\ 0 & 0.0045 & 0.0263 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 377 \\ -0.2779 & 0 & 0 & 0 & -0.3055 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 8000 & 0 & 0 & 0 \end{bmatrix}^T$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T$$

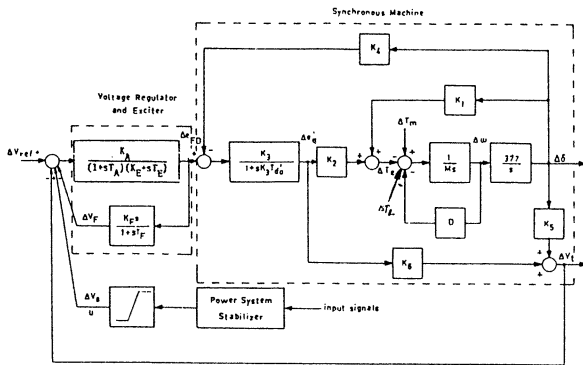


Fig. 4. Linearized incremental model of a synchronous machine with an exciter and stabilizer

A comparative study of the stabilization of power system will be made for four types of stabilizers: the conventional power system stabilizer, the linear optimal stabilizer, the PI stabilizer using the root locus method and the PI stabilizer using the suboptimal regulator approach. In all these cases, the dynamic responses following a 0.05 p.u. step change in the load demand ( $\Delta T_L = 0.05$ ) will be examined.

Case 1: The conventional power system stabilizer

The conventional power system stabilizer is [21]:

$$G(s) = \frac{U(s)}{\Delta\omega(s)} = \frac{20(1 + 0.017s + 0.02s^2)}{(1 + 0.05s)(1 + 0.05s)}$$

It should be noted that the stabilizer was designed by using eigenvalue shifting method [21]. Therefore, the overshoot in speed deviation would be somewhat greater. The tendency of great overshoot could be avoided if a least-square-error criterion is employed.

Case 2: The linear optimal stabilizer

The weighting matrices are chosen as follows:

$$Q_{11} = Q_{22} = Q_{33} = 10^7, Q_{44} = 10^5, Q_{55} = 10^7, Q_{66} = 0, R = 10^5$$

and all other elements of Q are zero. Note that Q and R must be selected such that the dominant eigenvalues are shifted to the left as far as possible [4] until the practical stabilizer limits are reached, that is,  $|u| \leq 0.12$ . The stabilizing signal is given by

$$U^*(t) = -GX(t)$$

where

$$G = [12.61 \quad 0.157 \quad 3.9 \times 10^{-3} \quad -0.935 \quad 4.66 \quad -309.3]$$

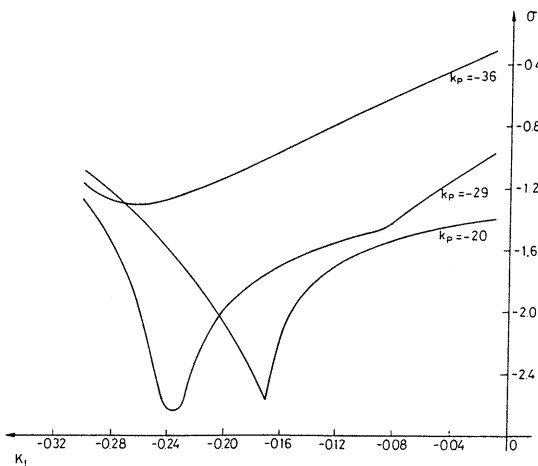


Fig. 5. Dominant eigenvalues for analog PI stabilizer (single machine system)

Case 3: The PI stabilizer by the root locus method

In this case, the real parts of dominant eigenvalue (the eigenvalue with minimum absolute value of real part) for various values of stabilizer gains  $K_P$  and  $K_I$  of the PI stabilizer is plotted in Fig. 5. An optimal set of stabilizer gains  $K_P = -29$  and  $K_I = -0.23$  is chosen.

Case 4: The PI stabilizer by the suboptimal regulator approach

The weighting matrices Q and R are chosen to be the same as those in case. 2. The stabilizer gains are found to be  $K_P = -19.24$  and  $K_I = -0.219$ .

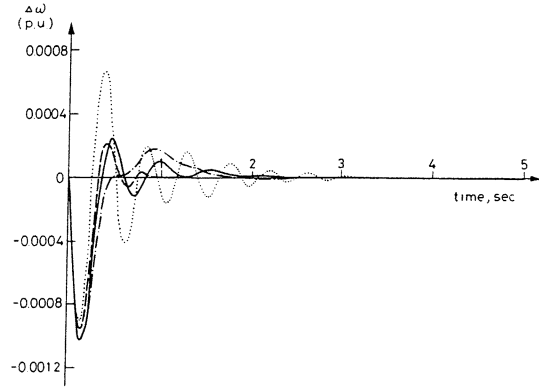


Fig. 6. Dynamic responses for analog PI stabilizer (single machine system)

- linear optimal stabilizer
- ..... conventional stabilizer
- PI stabilizer (root locus method)
- · - · - PI stabilizer (suboptimal regulator approach)

Fig. 6 shows the dynamic response of  $\Delta\omega$  when the system is subjected to a 0.05 p.u. step change in the load demand. A detailed comparison of the responses by using the four types of stabilizers is given in Table 1.

Table 1. Comparison of dynamic responses  $\Delta\omega$  for analog stabilizers (single machine infinite bus system)

	overshoot (p.u.)	settling time (sec)
linear optimal stabilizer	+0.0002 -0.00096	1.0
conventional power system stabilizer	+0.00068 -0.00090	3.5
PI stabilizer (root locus method)	+0.00024 -0.00104	2.0
PI stabilizer (suboptimal regulator approach)	+0.00018 -0.00104	2.4

It is found that the PI stabilizer yields better system performance than the conventional power system stabilizer in the sense of having less overshoot and shorter settling time. Therefore, the conventional power system stabilizer will not be considered again in the following discussions. For the PI stabilizer designed by the root locus method, the settling time is smaller while the overshoot is somewhat greater than that by the suboptimal regulator approach. The linear optimal stabilizer yields the best dynamic performance since all (six) state variables are employed as the feedback signals while only two state variables are required for the PI stabilizer. The linear optimal stabilizer is difficult for practical implementation since some state variables are not readily available.

**B. The Digital Power System Stabilizer**

For the digital power system stabilizer, a sampling period of 0.05 sec is chosen. The weighting matrices Q and R are selected to be the same as in the analog case. Three types of stabilizers are compared:

Case 1: The linear optimal stabilizer

In this case, the stabilizing signal is given by [20]

$$U^*(nT_s) = -G \times (nT_s) \quad n = 1, 2, \dots$$

where

$$G = [6.47, 0.104, 0.271 \times 10^{-2}, -0.944, 3.25, -110]$$

Case 2: The PI stabilizer by the root locus method

Fig. 7 gives the real parts of the dominant eigenvalues for the digital case. The optimal stabilizer gains are found to be  $K_P = -20$  and  $K_I = -0.17$ . The value of  $K_P = -25$  is not selected since the curve corresponding to  $K_P = -25$  is too sensitive with respect to the gain  $K_I$ .

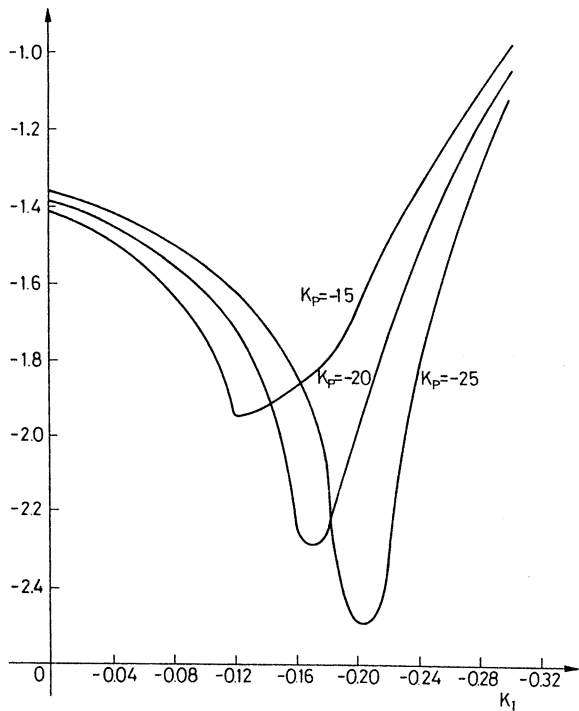


Fig. 7. Dominant eigenvalues for digital PI stabilizer (single machine system)

Case 3: The PI stabilizer by the suboptimal regulator approach  
The optimal stabilizer gains are

$$K_p = -16.37, K_I = -0.168$$

Fig. 8 shows the dynamic responses of  $\Delta\omega$  when the system is subjected to a 0.05 p.u. step change in the load demand. A detailed comparison of the responses by using the three types of stabilizers is given in Table 2.

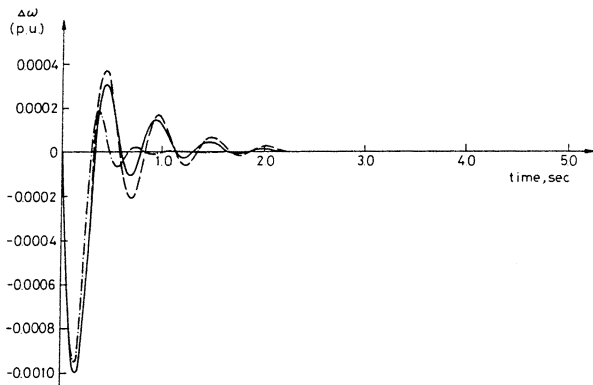


Fig. 8. Dynamic responses for digital PI stabilizer (single machine system)  
 - - - - - linear optimal stabilizer  
 - - - - - PI stabilizer (root locus method)  
 - . . . . PI stabilizer (suboptimal regulator approach)

Table 2. Comparison of dynamic responses  $\Delta\omega$  for digital stabilizers (single machine infinite bus system)

	overshoot (p.u.)	settling time (sec.)
linear optimal stabilizer	+0.00018 -0.00096	1.2
PI stabilizer (root locus method)	+0.00030 -0.0010	2.2
PI stabilizer (suboptimal regulator approach)	+0.00036 -0.0010	2.3

Just as the case of analog stabilizer, the linear optimal stabilizer yields better system dynamic responses than the PI stabilizer. However, the PI stabilizer is much simpler for practical implementa-

tion. It is also found that there is no significant difference between the responses obtained from using the two PI stabilizers.

### 5. EXAMPLE 2: MULTIMACHINE SYSTEM

Consider a 3-bus, 2 hydro-plants (10 machines each) system given in Fig. 9 [22]. Each machine is equipped with a fast exciter. The machine constants on a 100 MVA base, the transmission network data and the operating conditions are given in Tables 3-5.

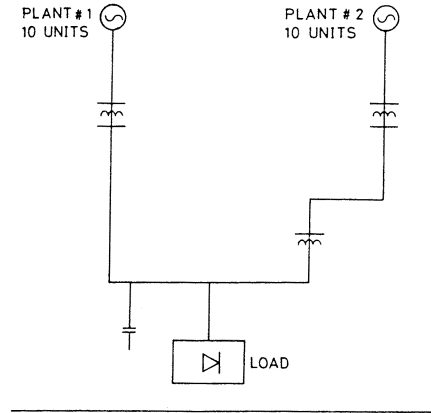


Fig. 9. System configuration for a multimachine power system

Table 3. Generator data

Plant No.	Unit No.	Rated power M	Inertia const. (sec)	Damping factor D (p.u.)	$T_{do}'$ (sec)	$X_d$ (p.u.)	$X_q'$ (p.u.)	$X_d'$ (p.u.)	$K_A$	$T_A$	$K$	$T_a$ (sec)	$T_b$ (sec)
(1)	10	100	4.1	1.0	4.1	0.921	0.535	0.106	500	0.02	0.153	0.17	0.5
(2)	10	100	4.042	1.0	6.1	0.678	0.4521	0.206	500	0.02	0.153	0.17	0.5

Table 4. Transmission network data

Line	Sending bus	End bus	Shunt admittance	Series admittance
1	1	3	0.0 + j0.0070	0.0005 + j0.0094
2	2	3	0.0 + j0.1080	0.0006 + j0.0361

Table 5. Operating conditions

Plant No.(1): 10 units, 598 MW, 282 MVAR, voltage= 1.0 /0 p.u.  
 Plant No.(2): 4 units, 340 MW, 187 MVAR, voltage = 1.03 /20° p.u.  
 Load bus: p = 936 MW, Q = 388 MVAR, |v| = 0.972 p.u.

Plant No.	$i_{do}$	$i_{qo}$	$i_{Do}$	$i_{Qo}$	$v_{do}$	$v_{qo}$	$v_{Do}$	$v_{Qo}$	$e_{qo}$	$\delta_o^\circ$
(1)	4.329	5.012	5.980	-2.826	0.268	0.963	1.000	0.000	1.052	15.5
(2)	2.677	2.629	3.398	-1.592	0.297	0.990	1.032	0.067	1.128	20.4

The state equation of the system can be written as

$$\dot{X} = AX + BU$$

$$Y = CX$$

where  $X = [\Delta\omega_1, \Delta\delta_1, \Delta e_{q1}', \Delta e_{FD1}, \Delta\omega_2, \Delta\delta_2, \Delta e_{q2}', \Delta e_{FD2}]^T$  is the state vector and  $U = [U_1, U_2]^T$  is the control vector. The system matrices A and B can be found in [22]. The output matrix C is given by

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

### A. The Analog Power System Stabilizer

A comparative study of the stabilization of multimachine power system will be made for five types of stabilizers: the linear optimal stabilizer, the two-level stabilizer [22], the global PI stabilizer, the local PI stabilizer by the root locus method and the local PI stabilizer by the suboptimal regulator approach. For comparison study, we use the same weighting matrices Q and R as proposed by Abdel-Magid [22]:

$$Q_{11} = Q_{55} = 1, \quad Q_{22} = Q_{66} = 10, \quad Q_{33} = Q_{77} = 1, \quad Q_{44} = Q_{88} = 0, \\ R_{11} = R_{22} = 1$$

and all the other elements are zero:

Case 1: The linear optimal stabilizer

$$\text{In this case,} \\ U = [U_1 \quad U_2]^T = G_O X$$

where

$$G_O = \begin{bmatrix} -149.2, & -2.796, & 0.774, & 2.37 \times 10^{-3}, & -24.72, & -0.628, & -0.106, & -2.2 \times 10^{-4} \\ -11.63, & -1.247, & -0.122, & -2.2 \times 10^{-4}, & -119.6, & -1.631, & 0.93, & 1.88 \times 10^{-3} \end{bmatrix}$$

Case 2: The two-level stabilizer [22]

For the two-level stabilizer, we have

$$U = G_t X$$

where

$$G_t = \begin{bmatrix} -140.4 & -2.458 & 0.721 & 0.022 & 0.0 & 0.016 & -0.159 & 0 \\ 0 & -0.027 & -0.409 & 0 & -121.7 & -1.63 & 0.931 & 0.0019 \end{bmatrix}$$

Case 3: The global PI stabilizer

Two types of PI stabilizers will be considered for the multimachine power system: the global PI stabilizer and the local PI stabilizer. The global PI stabilizer employs the speed signals of all machines in the system as the input. On the other hand, only the speed signal of the machine itself is required for the local PI stabilizer.

$$U = G_g X$$

$$G_g = \begin{bmatrix} -58.51 & -1.305 & 0 & 0 & -0.024 & -0.721 & 0 & 0 \\ -0.03 & -1.111 & 0 & 0 & -40.0 & -0.649 & 0 & 0 \end{bmatrix}$$

Case 4: The local PI stabilizer by the root locus method

For the local PI stabilizer, the procedure proposed in [7] is first used to determine the machine on which the stabilizer should be placed first. It is found [22] that machine 1 has lower frequency mode than machine 2. Therefore, it is decided that a local PI stabilizer should be first placed on machine 1. A second PI stabilizer is placed on machine 2 after the PI stabilizer on machine 1 has been installed. Following the same procedure as the case of single machine infinite bus system, it is found that the stabilizer gains of the local PI stabilizers are given by

$$K_{p1} = -58.5, \quad K_{I1} = -0.59 \\ K_{p2} = -40, \quad K_{I2} = -1.2$$

Case 5: The local PI stabilizer by the suboptimal regulator approach

$$K_{p1} = -98.71, \quad K_{I1} = -0.67 \\ K_{p2} = -25.23, \quad K_{I2} = -0.709$$

Fig. 10 shows the dynamic responses of  $\Delta\omega_1$  when the system is subjected to a 0.05 p.u. step change in load demand in machine 1. Notice that in Fig. 10 the responses obtained from using three types of stabilizers are included for comparison purposes: the linear optimal stabilizer, the two-level stabilizer and the global PI stabilizer. On the other hand, Fig. 11 gives the responses obtained from using the local PI stabilizers. A detailed comparison of the responses from using the various kinds of stabilizers is given in Table 6.

### B. The Digital Power System Stabilizer

Four types of stabilizers will be examined for comparison purposes: the linear optimal stabilizer, the global PI stabilizer, the local PI stabilizer by the root locus method and the local PI stabilizer by the suboptimal regulator approach.

Case 1: The linear optimal stabilizer

$$G_O = \begin{bmatrix} -63.33 & -0.951 & 0.053 & 4.69 \times 10^{-4} & -16.13 & -0.323 & -0.029 & -8.44 \times 10^{-5} \\ -22.22 & -0.881 & -0.063 & -2.48 \times 10^{-4} & -51.21 & -0.386 & 0.229 & 7.66 \times 10^{-4} \end{bmatrix}$$

Case 2: The global PI stabilizer

$$G_g = \begin{bmatrix} -60.52 & -0.867 & 0 & 0 & -18.79 & -0.306 & 0 & 0 \\ -28.85 & -0.858 & 0 & 0 & -38.0 & -0.142 & 0 & 0 \end{bmatrix}$$

Table 6. Comparison of dynamic responses  $\Delta\omega_1$  for analog stabilizers (multimachine system)

	overshoot (p.u.)	settling time (sec.)
linear optimal stabilizer	+0.00059 -0.00005	0.8
two-level stabilizer	+0.00088 -0.00034	1.4
global PI stabilizer	+0.00064 -0.00008	1.3
local PI stabilizer by the root locus method	+0.00069 -0.00002	1.3
local PI stabilizer by the suboptimal regulator approach	+0.00051 -0	1.6

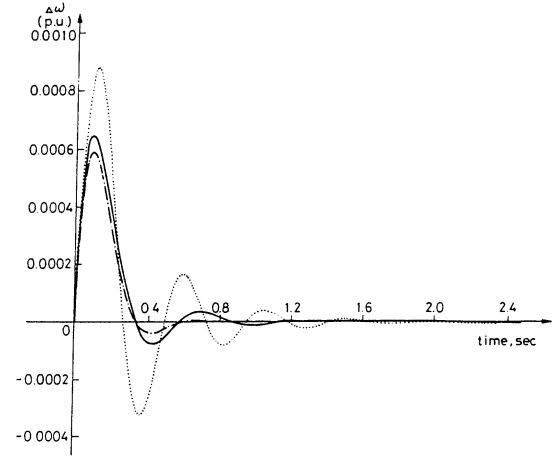


Fig. 10. Dynamic responses for analog PI stabilizer (multimachine system)  
 - - - - - linear optimal stabilizer  
 ——— global PI stabilizer  
 - - - - - two level stabilizer

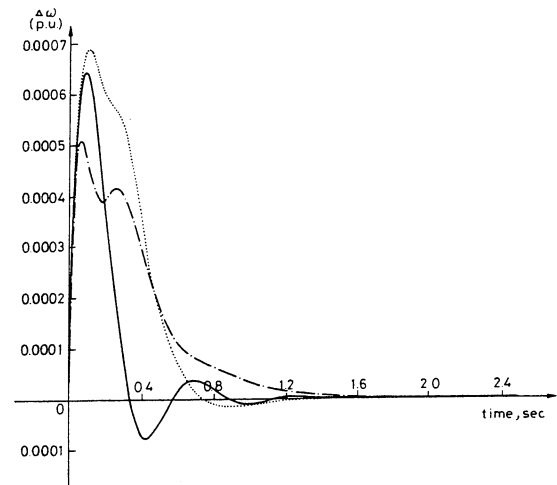


Fig. 11. Dynamic responses for analog PI stabilizer (multimachine system)  
 ——— global PI stabilizer  
 - - - - - local PI stabilizer (root locus method)  
 - - - - - local PI stabilizer (suboptimal regulator approach)

Case 3: The local PI stabilizer by the root locus method

$$K_{p1} = -48, \quad K_{I1} = -0.3 \\ K_{p2} = -10, \quad K_{I2} = -0.1$$

Case 4: The local PI stabilizer by the suboptimal regulator

$$K_{p1} = -81.25, \quad K_{I1} = -0.456 \\ K_{p2} = -25.2, \quad K_{I2} = -0.362$$

Fig. 12 shows the dynamic responses of  $\Delta\omega_1$  obtained from using the above digital stabilizers. A detailed comparison of the responses is given in Table 7.

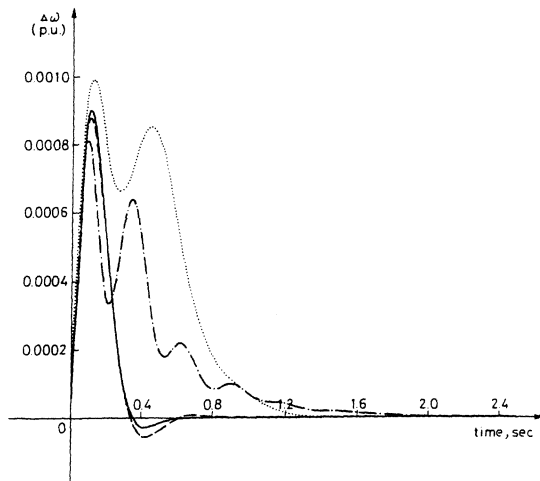


Fig. 12. Dynamic responses for digital PI stabilizer (multimachine system)  
 - - - - linear optimal stabilizer  
 - - - - global PI stabilizer  
 ····· local PI stabilizer (root locus method)  
 - · - · local PI stabilizer (suboptimal regulator approach)

Table 7. Comparison of dynamic responses  $\Delta\omega_1$  for digital stabilizers (multimachine system)

	overshoot (p.u.)	settling time (sec.)
linear optimal stabilizer	+0.00088 -0.00006	0.8
global PI stabilizer	+0.00089 -0.00003	0.8
local PI stabilizer by the root locus method	+0.00099 -0	1.3
local PI stabilizer by the suboptimal regulator approach	+0.00081 -0	1.9

From the simulation results in Figs. 10–12 and Tables 6–7, several observations for the multimachine systems are in order:

- (i) The linear optimal stabilizer yields the best dynamic performance. However, it is very difficult to implement such an optimal stabilizer since many states which are not readily available are required.
- (ii) The PI stabilizer yields better responses than the two-level stabilizer [22].
- (iii) Among the three kinds of PI stabilizers, the global stabilizer yields better responses than the others. However, four state variables including two of the other machine are required while only two state variables of the machine itself are required for the local PI stabilizers.
- (iv) On comparing the dynamic responses obtained by the two types of local PI stabilizers, it is found that the local PI stabilizer designed by the root locus method yields a response with shorter settling time while the one designed by the suboptimal regulator yields a response with less overshoot. This results from the fact that the dominant-eigenvalue criterion is employed by the root locus method while the least-square-error algorithm is employed by the suboptimal regulator approach.

## 6. CONCLUSIONS

A new technique for the stabilization of power systems is presented by using a proportional-plus-integral (PI) power system stabilizer. Two systematic approaches, i.e., the root locus method and the suboptimal regulator approach, have been proposed for the determination of the optimal stabilizer gains of the PI stabilizer. Being relatively simple for practical implementation, which is the reason why the PI controllers are widely used by the process industry,

is the main feature of the proposed PI stabilizer.

To demonstrate the effectiveness of the proposed PI stabilizer, two example power systems have been examined. Simulation results indicate that the PI stabilizer developed provides a new means for increasing the damping torque of the system and hence improving the dynamic stability of power systems.

The effect of PI stabilizer on power system transient stability limit, which is an important aspect in PSS design, is being investigated by the authors and the results will be presented in a forthcoming paper. Moreover, in order to obtain better dynamic performance for a practical power system whose operating conditions usually vary considerably, a self-tuning PI stabilizer with adjustable stabilizer gains based on adaptive control techniques [9, 13] is also under investigation by the authors.

## 7. ACKNOWLEDGEMENT

The authors would like to thank Mr. P.H. Hwang and Mr. W.G. Yang for their assistance in performing part of the computer simulations.

## 8. NOMENCLATURE

### General

A	system matrix in analog-mode
B	control matrix in analog-mode
X	state vector
Y	output vector
C	output matrix
U	control vector
$\phi$	system matrix in digital-mode
$\theta$	control matrix in digital-mode
J	performance index
Q	weighting matrix for state variables
R	weighting matrix for control signals
G	feed-back gain matrix
$\Delta$	linearized incremental quantity
S	Laplace operator

### System variables

$V_{ref}$	reference input voltage
$V_t$	terminal voltage
$V_o$	infinite bus voltage
$e_{FD}$	equivalent excitation voltage
$e'_q$	q-axis component voltage behind transient reactance
$V_F$	stabilizing transformer voltage
$V_S$	stabilizer output
$T_e$	energy conversion torque
$T_m$	mechanical input
$\delta$	torque angle
$\omega$	angular velocity

### System parameters

$r+jX_e$	tie line impedance
$G+jB$	terminal load admittance
$K_A, K_E$	voltage regulator gain
$T_A, T_E$	voltage regulator time constant
$K_F$	stabilizing transformer gain
$T_F$	stabilizing transformer time constant
$K_1 \sim K_6$	constants of the linearized model of synchronous machine
$T_{d0}$	d-axis transient open circuit time constant
M	inertia coefficient, $M=2H$
D	damping coefficient
$T_s$	sampling period

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### Discussion

**O. P. Malik** (The University of Calgary, Calgary, Alberta, Canada): The authors have presented an interesting approach to the design of a power system stabilizer. To better appreciate the contribution of the paper, the discussor would like the authors to clarify a number of points listed below.

1. The purpose of the power system stabilizer is to provide damping over a range of frequencies which in the case of most power systems is 0.5 to 2.0 Hz. Have the authors investigated the contribution provided by the proposed stabilizer algorithm over this frequency range?
2. There are in general 2 or 3 modes of oscillation in the 0.5 to 2.0 Hz range. It is not clear if the 'multimachine' power system model shown in Fig. 9 of the paper exhibits more than one mode of oscillation. The single machine - infinite bus model of Fig. 3 can in fact be compared to the 'multimachine' model of Fig. 9 if one considers that the two machines in Fig. 9 are different in size with one machine being much larger than the other. Such a model will exhibit only one mode of oscillation. Can the authors comment on this?
3. The authors describe the stabilizing signal  $U^*$  in Eq. (6) as optimal, whereas it is labelled as sub-optimal in Fig. 2(b). Which description is correct? Please clarify.
4. Case 4 in Section 4A refers to 'The PI stabilizer by the sub-optimal regulator approach', but this approach does not seem to be described anywhere in the paper. We are presented with the values of  $K_p$  and  $K_I$  without explaining how they are arrived at. Please explain.

5. Equations (8) and (9) have the same symbol on left hand side. Is it correct? Similarly, should Eq. (21) not read  $\phi_c^* = \phi^* + BF^8C$ ?
6. The statement in last paragraph of Section 4(A) that 'For the PI stabilizer designed by the root locus method, the settling time is smaller . . . than that by the sub-optimal regulator approach' does not seem to be borne by Fig. 6. In any case the magnitude of  $\Delta\omega$  being considered is so small, that it is insignificant and almost unmeasurable in practice. For example, can the authors measure  $\Delta\omega$  of 0.0754 rad/s? With these magnitudes the differences between the performances of cases 2, 3 and 4 can be ignored as they fall within the errors in the values of parameters used in these studies. Will the authors comment on the significance of their results in the light of these observations?
7. It would have been interesting to see the variation of  $\Delta\delta$  in view of the small magnitudes of  $\Delta\omega$ .
8. In going from Example 1 to Example 2, why is there such a large difference between the values of the Q and R matrices in the two cases?
9. Transmission network data in Table 4 shows two shunt inductive admittances and two series inductive admittances. However, in Fig. 4 (not showing any bus or line numbers), are load (capacitive or inductive?), one capacitor and three transformers are shown. Will the authors correlate the Table and the corresponding figure?
10. Please explain the process by which the second stabilizer is chosen in Case 4 of Section 5A.
11. It seems from the results given in the paper that the values of  $k_p$  and  $K_I$  depend upon the operating conditions of the system such as the number of units operating in each station. Does it mean that every time a unit is brought in or removed from the bus, the values of  $K_p$  and  $K_I$  will have to be adjusted to get good performance?

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**Yuan-Yih Hsu and Chung-Yu Hsu:** The authors express their appreciation to Dr. Malik for his valuable comments and we would like to clarify the question raised by the discussor as follows:

1. The effect of the proposed PI power system stabilizer on system damping can be more clearly demonstrated by a list of system eigenvalues. Table A gives the eigenvalues for the signal machine system and Table B lists those for the multimachine system. Only the eigenvalues for the analog stabilizers are presented here.

Table A Eigenvalues for single machine system

stabilizer type	eigenvalues in s-plane			
open-loop	-0.235 ± j0.8	-1.55	-3.08	-8.12 ± 8.92
conventional	-1.480 ± j13.3	-1.58 ± j0.53	-2.56 ± j9.13	-25.1 ± j10.2
optimal	-1.0	-3.07 ± j33.2	-4.64 ± j12.0	-8.10
root-locus	-2.61 ± j2.97	-2.64	3.41 ± 12.7	-6.67
suboptimal	-1.78 ± j1.18	-2.66 ± j11.7	-6.24 ± j5.93	

Table B Eigenvalues for multimachine system

stabilizer in	eigenvalues in s-plane				
open loop	-0.0006	-0.09 + j9.84	-0.244	-25.17 ± j67.8	-25.23 ± j30.3
plant 1	-3.94 ± j7.13	-4.05 ± j6.21	-20.8 ± j66.2	-21.8 ± j28.4	
plant 1 & 2	-5.39 ± j4.40	-7.93 ± j23.4	-17.0 ± j13.7	-20.3 ± j66.2	

2. It is not proper to replace the multimachine power system in Section 5 by a single machine-infinite bus system since the two plants are of the same rating.
3. The stabilizing signal  $U^*$  in eqn. (6) is optimal in the sense of minimizing the performance index given eqn. (5). The term "suboptimal regulator" is employed in Fig. 2 since only the output variable instead of all the state variables is used as the input signal to the stabilizer.
4. The values of  $K_p$  and  $K_I$  were obtained by using a Newton-Raphson iterative algorithm based on eqns. (7) through (10) (see Reference A).